

25.  An advertisement for real estate published in the 28 July 2004 electronic edition of the *New York Times* states:

Did you know that the percent increase of the value of a home in Manhattan between the years 1950 and 2000 was 721%? Buy a home in Manhattan and invest in your future.

Suppose that instead of buying a home in Manhattan in 1950, someone had invested money in a bank account that compounds interest four times per year. What annual interest rate would the bank have to pay to equal the growth claimed in the ad?

Solution: An increase of 721% means that the final value is 821% of the initial value. Let r denote the interest rate the bank would have to pay for the 50 years from 1950 to 2000 to grow to 821% of the initial value. At that interest rate, compounded four times per year, in 50 years an initial amount of P dollars grows to

$$P\left(1 + \frac{r}{4}\right)^{4 \times 50}$$

dollars. We want this to equal 8.21 times the initial amount, which means that we need to solve the equation

$$P\left(1 + \frac{r}{4}\right)^{200} = 8.21P.$$

To solve this equation, divide both sides by P and then raise both sides to the power $1/200$, getting

$$1 + \frac{r}{4} = 8.21^{1/200}.$$

Now subtract 1 from both sides and then multiply both sides by 4, getting

$$r = 4(8.21^{1/200} - 1) \approx 0.0423.$$

Thus the annual interest would need to be approximately 4.23% to equal the growth claimed in the ad.

Note that 4.23% is not a particularly high return for a long-term investment, contrary to the ad's implication.