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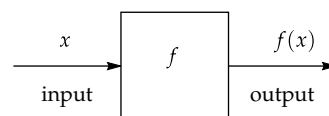
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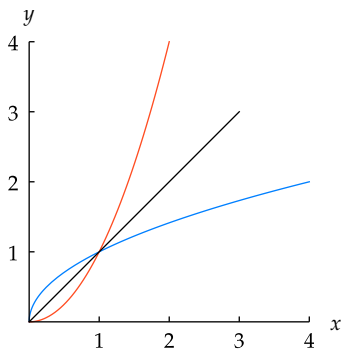
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The graphs of x^2 (orange) on $[0, 2]$ and its inverse function \sqrt{x} (blue) on $[0, 4]$ are symmetric about the line $y = x$.

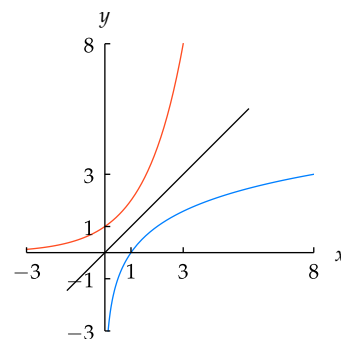
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The graphs of 2^x (orange) on $[-3, 3]$ and its inverse function $\log_2 x$ (blue) on $[\frac{1}{8}, 8]$ are symmetric about the line $y = x$.



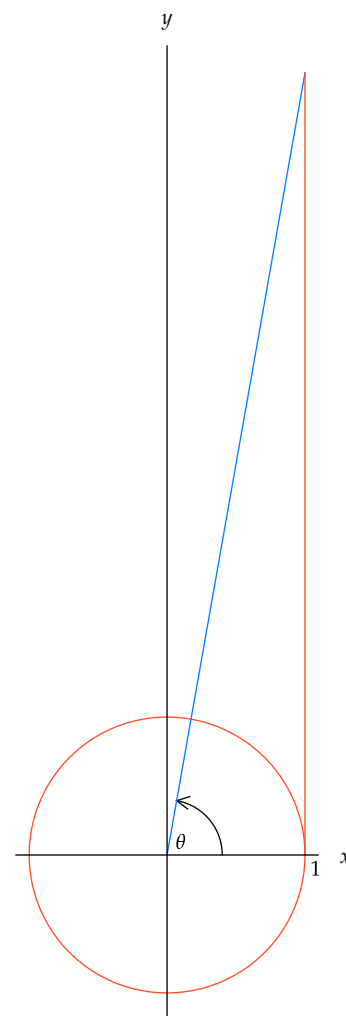
STARRY NIGHT, painted by Vincent Van Gogh in 1889. The brightness of a star as seen from Earth is measured using a logarithmic scale.

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The blue line segment has slope $\tan \theta$. The orange line segment has length $\tan \theta$.



The Greek mathematician Hipparchus, depicted here in a 19th-century illustration, developed trigonometry over 2100 years ago as a tool for calculations in astronomy.

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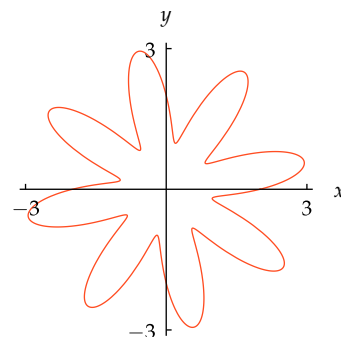
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The graph of the polar equation $r = 2 + \sin(8\theta)$ for θ in $[0, 2\pi]$.